Chordal Graphs

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Outline

1 Chordal Graphs: Definition

2 Chordal Graphs: Recognition

3 Chordal Graphs: Clique Tree Representation

Chordal Graphs

A chord in a cycle is an edge which goes between two vertices which are not consecutive in the cycle.

A graph G is chordal if there are no chordless cycles in G of length greater than three.

Chordal graph always contain a vertex v such that the neighborhood of v is a clique. Such a vertex is called a simplicial vertex.
A perfect elimination scheme $v_1, v_2, v_3, \cdots, v_n$ is an ordering of the vertex set if and only if for all i, v_i is simplicial in the graph induced by v_{i+1} through v_n.

Number of Chordal Graphs

The number of chordal graph is $2^{\Omega(n^2)}$.

Proof. Chapter 15

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Recognition of Chordal Graphs

Trivial algorithm $O(n^4)$ time.

Recognition of Chordal Graphs

Trivial algorithm $O(n^4)$ time.
G is chordal if and only if there is a perfect elimination scheme for G.

Linear Algorithm:

- Construction phase: a construction phase which creates an ordering which is a perfect elimination scheme if and only if G is chordal.
- Verification phase: a verification phase which checks whether the ordering which was constructed is infact a perfect elimination scheme.

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Clique Tree Representation

Chordal graphs correspond exactly to intersection graphs of subtrees of a tree. This model is called clique tree model. A tree T is a clique tree of a graph G if the nodes of T correspond to maximal cliques of G and each vertex v of G corresponds to a subtree of cliques which contain v.

Construction Clique Tree Representation

Assume that vertices are labeled from 1 to n according to their position in a perfect elimination scheme.

We construc the clique tree for the graph induced on vertices i through n for all vertices, starting with $i = n$ and ending with $i = 1$.

Let $C(v)$ be the clique consisting of v and all neighbors of v which appear after v in the elimination scheme. After each vertex v is processed, v is given a pointer to $C(v)$. Note that vertices may be added to this clique later in the algorithm, but v will always point to a clique which contains $C(v)$.

Let i be the next vertex considered, and assume we know the clique tree on the graph induced by $i+1, \cdots, n$. We need to add $C(i)$ to the clique tree.

Construction Clique Tree Representation (Contd.)

Let i be the next vertex considered, and assume we know the clique tree on the graph induced by $i+1, \cdots, n$. We need to add $C(i)$ to the clique tree.
Let j be the first vertex of $C(i)$ in the elimination ordering, other than i itself. If $|C(i)| = 1 + |C(j)|$, and the clique pointed to by j is equal to $C(j)$, we add i to this clique. Otherwise, add $C(i)$ as a new node of the tree. Connect $C(i)$ to the tree by adding an edge from $C(i)$ to the clique pointed to by j.