Graph Representation

Md. Saidur Rahman

Department of Computer Science and Engineering,
Bangladesh University of Engineering and Technology, Dhaka

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Outline

Traditional Representation of Graphs

- Adjacency Matrix
- Adjacency List

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Space Complexity

- Adjacency Matrix $O(n^2)$
- Adjacency List $O(n+m)$

Efficient Representation of Graphs

In terms of bits?

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- Adjacency List $\Theta(n^2 logn)$ bits

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How many bits are required to represent a graph of n vertices?

- The number of graphs on n vertices is $2^{n(n-1)/2}$.
- Any form of representation for the class of all graphs must use at least $n(n-1)/2$ bits.
- Since every form of representation must use $\Omega(n^2)$ bits, and adjacency matrices use $O(n^2)$ space, adjaceny matrices come within a constant factor of the minimum possible amount of space.

Space optimal representation

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- Thus adjacency matrices are a space optimal representation of graphs.
- Adjacency lists are not space optimal.

Counting Labeled and Unlabeled Graphs

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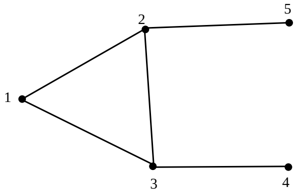
Unlabeled graphs?

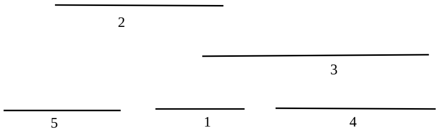
- We need to consider isomorphic copies. There are at most $n!$ isomprphic copies of an unlabeled graph.
- Thus the number is $2^{n(n-1)/2}/n!$.

Outline

Interval Graphs

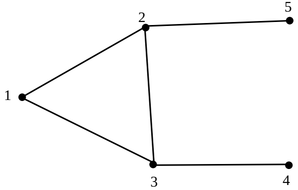
In an interval graph, vertices correspond to intervals on real line, and two vertices are adjacent if and only if the corresponding intervals have a nonempty intersection.

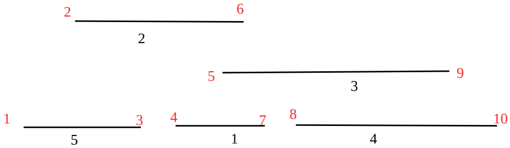




Representation of Interval Graphs

Label the endpoints of intervals in increasing order of their appearance on the line. This assigns to every vertex two integers in the range $1 \cdots 2n$. For each vertex v, we store the labels of the endpoints of v; this uses $O(logn)$ bits per vertex. Total space required is $O(nlogn)$.



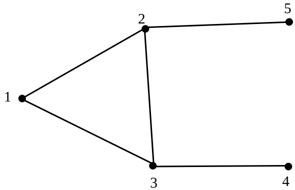


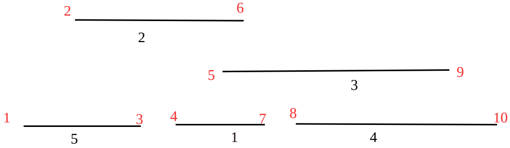
1: 4, 7
2: 2, 6
3: 5, 9
4: 8,10
5: 1, 3

Representation of Interval Graphs

Adjacency query

Are two vertices *x* and *y* adjacent?





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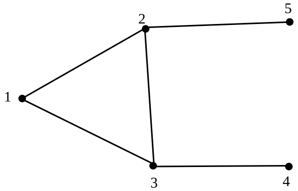

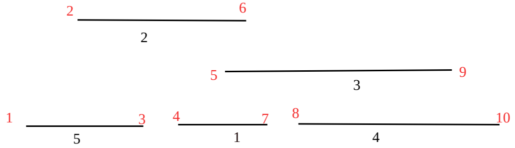

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Can be determined easily from the labels of *x* and *y* in constant time.